Non-Constant Momentum Compaction

- Dependence of traversal time on energy:

$$T = T_0 + LA_{56}\Delta + \frac{1}{2}LA_{566}\Delta^2$$

  - L is arc length
  - T_0 is traversal time for reference energy
  - Δ is energy difference from reference energy
  - η (frequency slip factor) is linearly related to

$$A_{56} + A_{566}\Delta$$

  - η difficult to keep independent of energy
- What is the effect on the RF bucket?
  - Area
  - Stability

Hamiltonian Computataion

- The Hamiltonian (smoothed)

$$-\frac{1}{2}A_{56}\Delta^2-\frac{1}{6}A_{566}\Delta^3+\frac{v}{\omega}[\sin(\omega\tau+\phi)-\omega\tau\cos\phi-\sin\phi]$$

- Bucket edges:
  - $A_{566}=0$

$$\Delta=\pm\Delta_0(\omega\tau)=\pm\sqrt{\frac{2v}{A_{56}\omega}}\sqrt{\sin(\omega\tau+\phi)+\sin\phi-(\omega\tau+2\phi)\cos\phi}$$

  - A_{566} small (perturbation theory)

$$\Delta=\pm\Delta_0(\omega\tau)-\frac{A_{566}}{6A_{56}}\Delta_0^2(\omega\tau)$$

    - Note both top and bottom edges shift by same amount, direction
    - Area about same

- Large A_{566}: edge breaks eventually. For stability

$$|A_{566}| < \sqrt{\frac{A_{56}^3\omega}{3v(\sin\phi - \phi\cos\phi)}} = |A_{56}|\,\frac{2}{\sqrt{3}}\frac{1}{\Delta_0(0)}$$

  - ⋆ Stay away: nonlinearities destroy unstable fixed point
  - ⋆ Variation of η over bucket: $-\eta$ to 2η

Conclusions

- A_{566} irrelevant over reasonable ranges
- A_{5666} may be more important